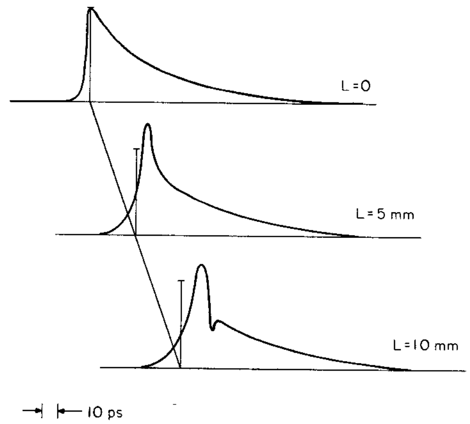

Fig. 6. Propagation of an asymmetric pulse for the example microstrip line.

frequency components which travel faster and catch up with the high frequency components in the rising edge.

V. Conclusion

Dispersion in a microstrip line has been analyzed numerically. The calculated output pulse shapes show how the pulses are distorted in the time domain. These give ideas in what to expect in picosecond electrical pulse generations. The approximate analytical formula shows the separation of the high and low frequency components and is useful in a qualitative sense. Application of this analysis to a pulse with practical parameters shows an interesting sharpening effect of the dispersion.

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The Equivalent Circuit of the Asymmetrical Series Gap in Microstrip and Suspended Substrate Lines

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Abstract —**The microwave properties of the series gap in microstrip and suspended substrate lines with unequal widths of the involved lines are described by means of suitable equivalent circuit data. These data have been computed using a rigorous three-dimensional spectral domain hybrid-mode approach developed by Jansen for the numerical characterization of the frequency-dependent scattering parameters of a wide class of strip and slot discontinuities. The results presented extend considerably the range of published gap data. In particular, they show that the stray-susceptances in the equivalent pi-network of the asymmetric series gap exhibit an inductive behavior for the case of tight coupling.**

Manuscript received January 8, 1982; revised April 5, 1982.
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I. Introduction

Due to its importance for the end-to-end coupling of resonators in stripline and microstrip measurement configurations and filter structures, the strip series gap and its equivalent circuit have been treated repeatedly in the microwave literature of the past years (see, for example, references [1]–[11]). With the exception of a few selected results recently presented by the authors of this paper [12], the gap data available, however, have been derived by quasi-static computational methods or by measurements and are all restricted to the symmetric gap, i.e., the gap between the ends of strips of equal width [13]. For this type of symmetric strip discontinuity, it is commonly accepted that it can be modeled by a purely capacitive equivalent pi-circuit, independent of the degree of coupling, at least as long as the gap dimensions are small compared to wavelength. Physically, this means that the effect of the stray field at the ends of the coupled strips is dominant compared to the inductive current density disturbance created there, if strips of equal width are considered.

The paper presented here directs attention to the asymmetric series gap discontinuity which is shown to behave differently. The equivalent circuit data given in this contribution have been computed employing a rigorous three-dimensional spectral domain approach developed by Jansen [14] and recently proposed by Jansen and Koster as a unified CAD basis for the frequency-dependent characterization of MIC components [12]. The numerical approach used reduces the solution of the gap hybrid-mode problem to the satisfaction of electromagnetic boundary conditions in a small region around the gap. All other boundary conditions—for example, those on the exciting microstrip lines—can be satisfied in advance in preliminary phases of the computation, as is described in detail in [14]. The two-dimensional integral operator equation representing the boundary value problem is solved by Galerkin's method [12],[14].

It should be noted explicitly that the results achieved by the before-mentioned numerical approach are in terms of frequency-dependent scattering parameters and do not rely on the assumption of a special kind of equivalent circuit. Nevertheless, although the information inherent in scattering parameters satisfies the requirements of circuit design completely, there is a definite need for equivalent circuit data since in design considerations the use of these is much more convenient to microwave engineers. Moreover, suitably chosen equivalent circuits have the advantage that they are tightly related to physical interpretation and, therefore, numerically computed scattering parameters have been transformed into equivalent pi-circuit elements in this paper. The reference impedances used for the transformation, i.e., the characteristic impedances of the involved strip transmission lines, are also given for reasons of clarity and uniqueness. For the limiting case of equal widths, the computer program used here has been tested successfully by comparison with measurements and the results provided by other authors (see [12]).

II. Results

The numerical results reported have been generated on a Control Data Cyber 76 computer which is installed at the University of Cologne computer center. Their accuracy, with respect to converged values, is estimated to be in the order of magnitude of 2 percent, at least as far as the elements are concerned which dominate the behavior of the respective equivalent circuit. However, it should be clear that accuracy is lost in the case of very loose gap coupling for the coupling element and in the case of small gap width for the stray element of the narrower line encountered. The reported data all refer to the geometry of the asymmetrical gap formed between the ends of strip transmission lines on a two-layer substrate as depicted in Fig. 1. In the microstrip case, the height h_l at which the carrier sheet of dielectric constant ϵ_s is thought to be suspended above the bottom ground plane is reduced to $h_l = 0$.

Fig. 1(a) shows the planar strip metalization pattern defining the geometry of the asymmetrical series gap on the surface of the substrate carrier sheet. The metalization is assumed to have zero thickness and infinite conductivity. The configuration involves a lateral shielding of width $s = \max(w_1, w_2) + 10h_s$ that can be expected to have only a weak influence on the electrical properties of the gap two-port, the reference planes of which are located at the ends of the strips of unequal widths. In Fig. 1(b), a longitudinal-section view of the structure under consideration is given. The dielectric constants ϵ_l and ϵ_u of the layers of media below and above the substrate carrier sheet (ϵ_s) are assumed to be $\epsilon_l = \epsilon_u = 1$. The distance of the top shielding at height h_u above the plane of metalization is set to have only negligible influence, i.e., $h_u = 10h_s$. If the case of suspended substrate lines is considered, $h_l = h_s$ prevails. Fig. 1(c) shows the equivalent pi-circuit associated with the described thin-film gap structure and its embedding between the adjacent transmission lines.

It should be clear that the gap equivalent pi-circuit of Fig. 1(c) consisting of a gap capacitance C_g and the stray susceptances jB_{s1} and jB_{s2} is only a formal representation of the complex frequency-dependent hybrid-mode electromagnetic field excited in the surrounding of the end-to-end coupled lines. Therefore, the gap equivalent circuit elements are frequency-dependent quantities.

However, it is well known that the equivalent circuit parameters of the symmetric gap are only weakly dependent on the operating frequency up to about 18 GHz on the common commercially used substrates. From investigations made by Easter, Gopinath, and Stephenson [8], it can be concluded that changes with frequency in the gap equivalent length, similar to the open-end effect, are comparable with measurement errors within the range up to 18 GHz. Rizzoli and Lipparini [11] have determined the frequency dependence of the equivalent capacitancies of the symmetric microstrip gap explicitly by accurate resonance measurements. Their results also show that gap capacitances indeed exhibit only a weak variation with frequency up to 18 GHz. Furthermore, the theoretical symmetric gap data given by Jansen and Koster [12] in a previous paper also confirm the fact that the change with frequency of the gap parameters is not very strong, even if frequency dependent characteristic microstrip impedance is taken into account in contrast to [11] where a static characteristic impedance is used. Finally, all these results have again been confirmed in principle by the computations made here for the characterization of the asymmetrical series gap, even if the extent of frequency dependence is, of course, a function of the specific geometry considered. For this reason, the results presented have all been limited to a single operating frequency, namely $f = 4$ GHz.

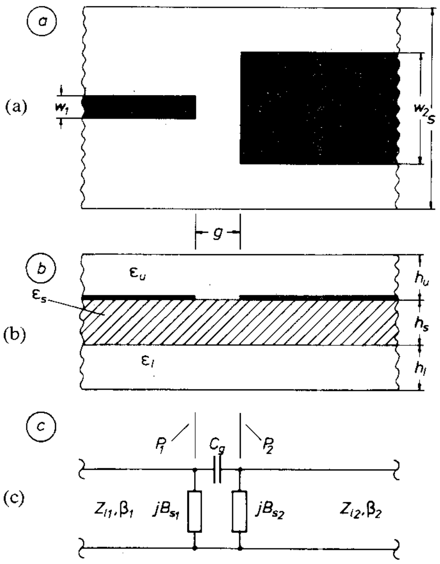


Fig. 1. (a) Planar strip metalization pattern, (b) longitudinal-section view, and (c) equivalent pi-circuit of the asymmetrical series gap.

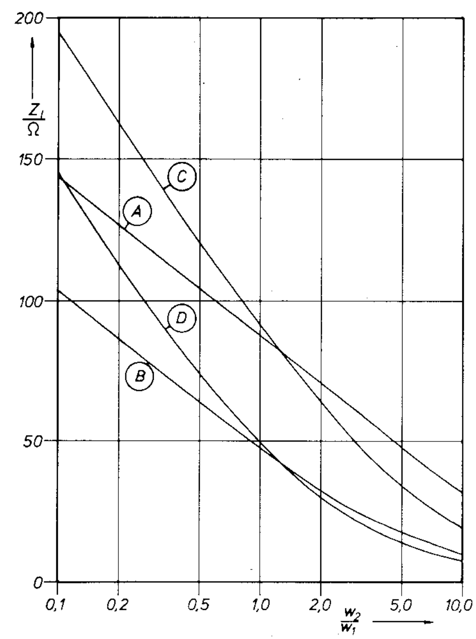
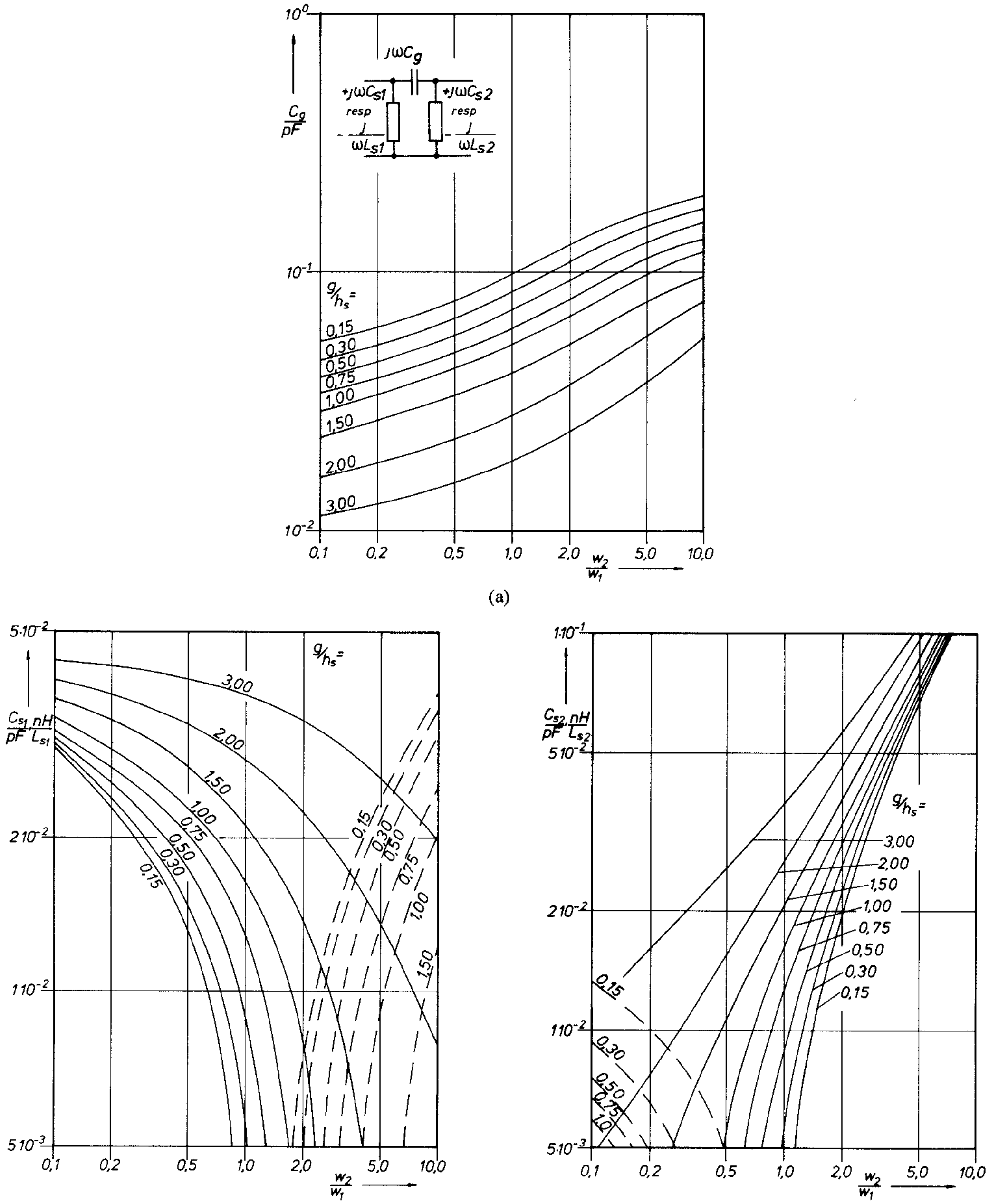


Fig. 2. Characteristic impedances used for the conversion of scattering parameters into equivalent circuit elements ($f = 4$ GHz). A: $\epsilon_s = 10.4$, $h_s = 0.635$ mm, $h_l = h_s$, $w_1 = 0.635$ mm (see Fig. 3). B: $\epsilon_s = 10.4$, $h_s = 0.635$ mm, $h_l = 0.$, $w_1 = 0.635$ mm (see Fig. 4). C: $\epsilon_s = 2.35$, $h_s = 0.635$ mm, $h_l = 0.$, $w_1 = 0.635$ mm (see Fig. 5). D: $\epsilon_s = 2.35$, $h_s = 0.790$ mm, $h_l = 0.$, $w_1 = 2.360$ mm (see Fig. 6).

The characteristic impedances used for the conversion of scattering parameters into equivalent circuit element values in this paper are given in Fig. 2. They are generated in the gap computer program as a by-product of the numerical algorithm [14]. The definition of characteristic impedance employed is based on the electromagnetic power transported along the considered strip transmission lines in conjunction with the associated longitudinal strip current (see, for example, [15] and [16]). With the knowledge of these impedances available, it is possible to recalculate in a unique way the gap scattering parameters from



(b) (c)

Fig. 3. Equivalent pi-circuit elements of asymmetrical gaps in suspended substrate line at $f = 4$ GHz. $\epsilon_s = 10.4$, $h_s = 0.635$ mm, $h_1 = h_s$, $w_1 = 0.635$ mm. (a) Coupling capacitance. (b) Stray element on side 1. (c) Stray element on side 2. ——— capacitive; - - - inductive.

which the equivalent circuit data given here were derived.

Computed element values of the equivalent circuit of asymmetrical series gaps are given in Figs. 3–6 for several representative cases which will be specified in detail below. In all of the configurations investigated, the width w_1 of one of the strips is kept constant at $w_1 = h_s$, respectively, $w_1 \cong 3h_s$, with h_s denoting the thickness of the substrate carrier, whereas the width w_2 of the other strip transmission line involved is varied between $w_2/w_1 = 0.1$, and $w_2/w_1 = 10.0$. Also, a wide range of gap spacings g is considered and the value of g serves as a parameter of the curves presented. In this way, the electrical data of the asymmetrical series gap are provided as a function of the planar geometry. In order to allow for an accurate reading of the graphs despite the wide range of widths and spacings discussed, a double logarithmic scale has been chosen. Nevertheless, although the numerical values associated with the cases examined in Figs. 3–6 are quite different, the general tendency of the curves depicted there is the same. Therefore, most of the characteristic features describing the gap behavior can be discussed beginning with the suspended substrate case of Fig. 3. Further insight into the gap behavior is then provided by the subsequent discussion of Figs. 4–6.

In Fig. 3(a) the gap coupling capacitance C_g is graphically displayed as a function of strip width w_2 and gap spacing g for the asymmetric series gap between suspended substrate lines on a substrate of dielectric constant $\epsilon_s = 10.4$ and thickness $h_s = 0.635$ mm (RT-Duroid 6010). Starting from low values, the gap capacitance C_g increases monotonically with increasing values of w_2. In the limit of large widths w_2, the function $C_g(w_2)$ shows an

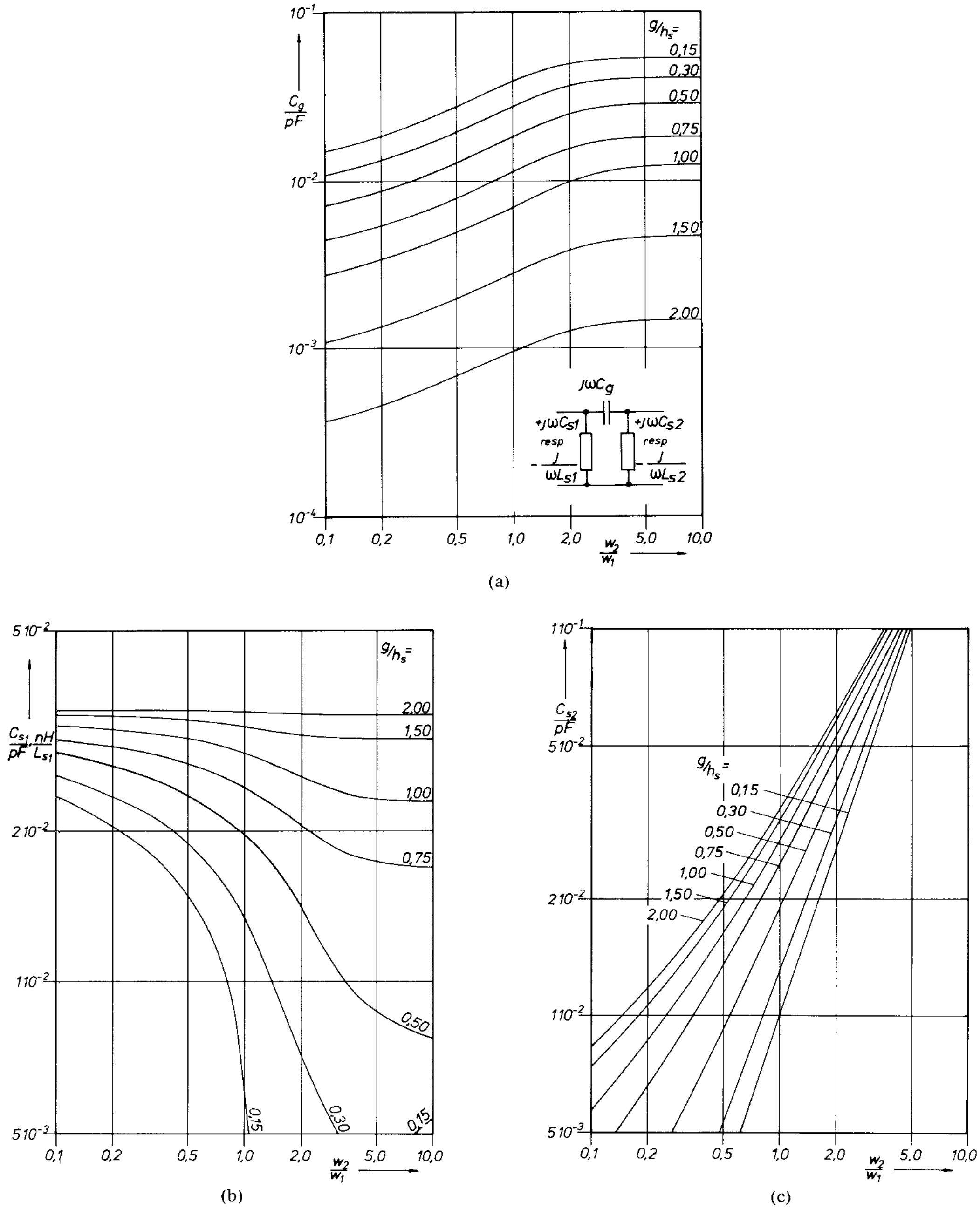


Fig. 4. Equivalent pi-circuit elements of asymmetrical gaps in microstrip line at $f = 4$ GHz. $\epsilon_s = 10.4$, $h_s = 0.635$ mm, $h_1 = 0.$, $w_1 = 0.635$ mm. (a) Coupling capacitance. (b) Stray element on side 1. (c) Stray element on side 2. ——— capacitive, - - - inductive.

asymptotic behavior and saturates at values which for the case under consideration are in the order of magnitude of approximately 0.1 pF. This is understandable since the electric field emerging from the end of strip 1, which has a constant width of w_1, can interact essentially only with that fraction of the strip width w_2 which is positioned in the direct neighborhood opposite to the end of line 1. The concentration of the electric field to the surrounding of the end of line 1 is pronounced for tight coupling, i.e., small gap spacings g. Therefore, if the value of g is small the slope of the curves $C_g(w_2)$ has a maximum near $w_2/w_1 = 1$.

For larger gap spacings g, this maximum is shifted to higher values of w_2 as a result of spreading of the end-to-end coupling field. The saturation-like behavior of the curves is also present for small widths w_2 of the encountered strip 2. This is due to the logarithmic scale and the fact that there is interaction even with a strip of vanishing width w_2. The reason for the increase of the coupling capacitance with vanishing gap spacing g should be obvious from quasi-static physical considerations. The numerical value of C_g varies by about one order of magnitude, i.e., a factor of 10, in the range of technologically and practically meaningful gap spacings g.

In Fig. 3(b), the results computed for the stray element value associated with the end of strip 1 of constant width w_1 (suspended substrate lines on RT-Duroid 6010) are represented as a function of the variables w_2 and g.

The capacitive behavior of the stray susceptance assigned to the end of strip 1 changes to an inductive one if the width of strip 2 increases more and more. The change-over to inductive values

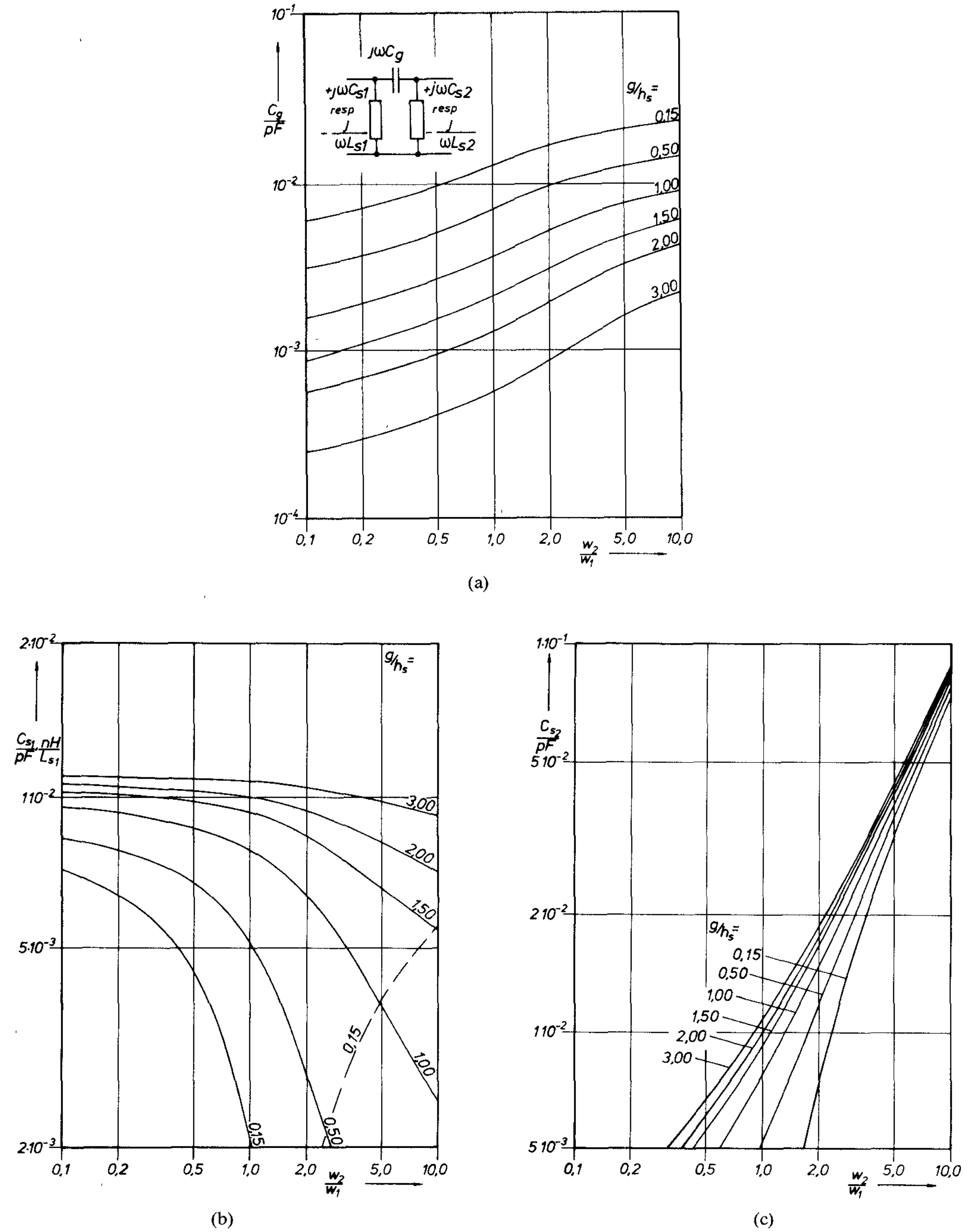


Fig. 5. Equivalent pi-circuit elements of asymmetrical gaps in microstrip line at $f=4$ GHz. $\epsilon_s=2.35$, $h_s=0.635$ mm, $h_1=0.$, $w_1=0.635$ mm. (a) Coupling capacitance. (b) Stray element on side 1. (c) Stray element on side 2.——— capacitive;- - - inductive.

can be seen to occur for linewidth ratios slightly higher than $w_2/w_1=1$, in the case of tight coupling and at higher values of w_2, if the gap spacing g is larger.

This justifies the interpretation that the asymmetric series gap exhibits some characteristics of an impedance step at the same time. Indeed, in the limit of vanishing gap spacings g and with the impedance of the gap capacitance C_g approaching a series short circuit at microwave frequencies this is understandable and should be expected. The asymmetry of the structures under investigation is responsible for the interesting effect that the current density disturbance created near the end of the narrow strip involved may supersede the effect of the open-end electric stray field, especially for tight coupling and a high degree of asymmetry. The same is obvious from Fig. 3(c) for the stray element which can be assigned to the end of the other strip of variable width w_2, however, in a much less accented way. In both cases, Fig. 3(b) and Fig. 3(c), the inductive element is present on that side of the asymmetric gap which is characterized by the smaller width of the two strips incorporated. This is in agreement with the microwave behavior of impedance steps where the larger inductive contribution can be associated with the line of smaller strip width, too [13]. If coupling is loose, Fig. 3(b) and (c) are also in agreement with physical understanding. Then, the stray capacitance C_{s1} of Fig. 3(b) saturates near the open-end value of strip 1. Furthermore, the stray capacitance C_{s2} of strip 2 increases linearly with the strip width w_2 as can be seen from Fig. 3(c).

Fig. 4(a)–(c) refers to the microstrip case and the same substrate (RT-Duroid 6010, $\epsilon_s=10.4$, $h_s=0.635$ mm) as that prevail-

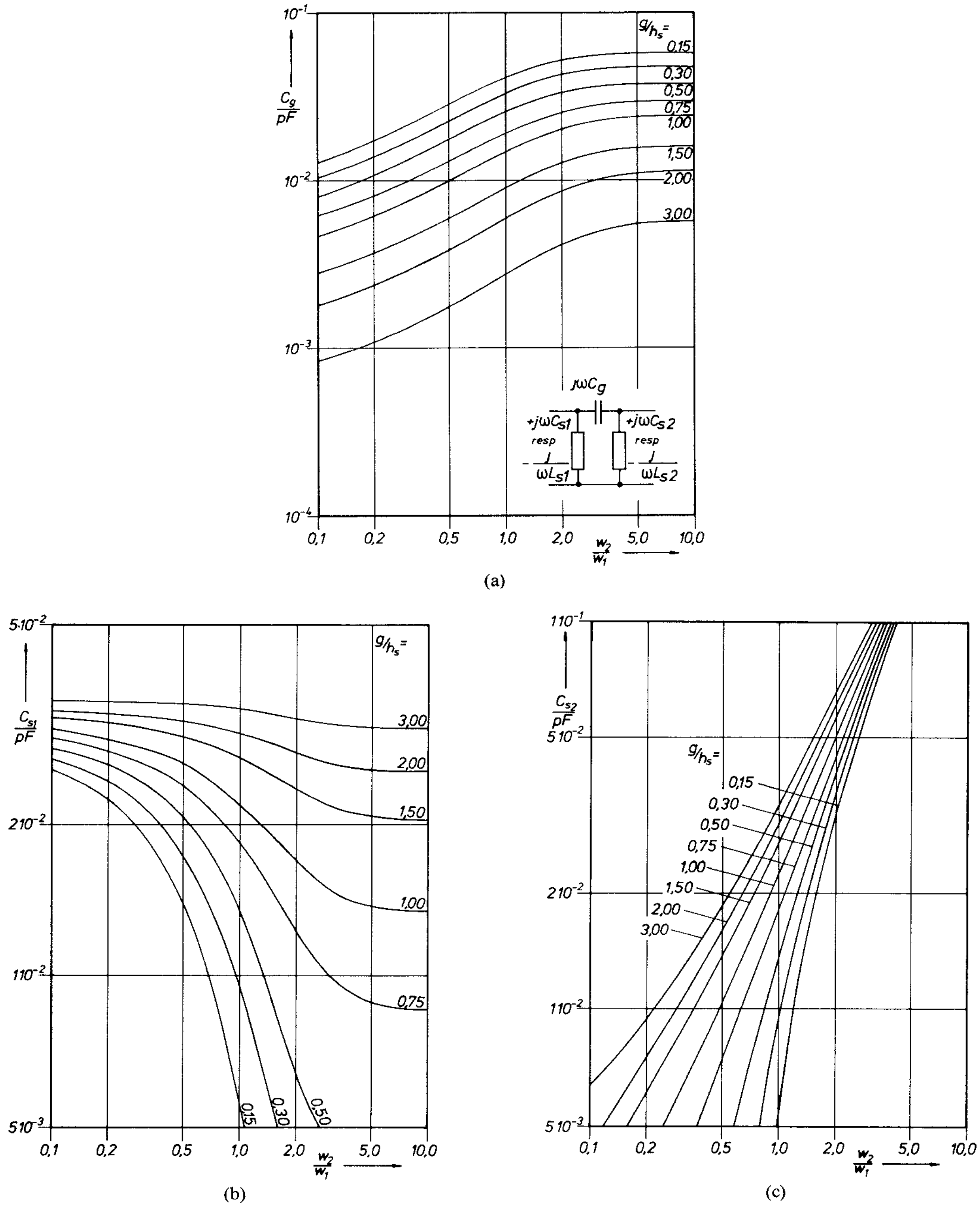


Fig. 6. Equivalent pi-circuit elements of asymmetrical gaps in microstrip line at $f=4$ GHz. $\epsilon_s=2.35$, $h_s=0.790$ mm, $h_1=0$, $w_1=2\ 360$ mm. (a) Coupling capacitance. (b) Stray element on side 1. (c) Stray element on side 2.

ing for the suspended substrate configuration of Fig. 3. As a consequence of higher concentration of the electric field in the substrate below the microstrip lines considered, the resulting gap series capacitances C_g of Fig. 4(a) have lower values compared to their suspended substrate counterparts of equal planar geometry. For the same reason, the change-over of the stray element C_{s1} to inductive values L_{s1} hardly becomes visible in Fig. 4(b) except for a very high degree of asymmetry near $w_2/w_1=10.0$ and only for the smallest value of gap spacing $g=0.15h_s$. For the lowest degree of coupling considered, i.e., for $g=2h_s$, the stray capacitance C_{s1} associated with the end of strip 1 stays approximately constant near its open-end effect value independent of the change of width of strip 2. From the curvature of the function $C_{s1}(w_2)$ with different values of g, it can be concluded that even for a high asymmetry of widths the inductive effect of the respective stray-field is over compensated by the capacitive end-effect unless the gap spacing reduces to $g<0.5h_s$. As can be seen from Fig. 4(c) and the results of Fig. 3, the stray element representing the field disturbance at the end of strip 2 itself can be modeled as a capacitance of value C_{s2} which increases nearly proportional to the strip width w_2.

The effect of a lowering of the dielectric constant can be extracted from Fig. 5(a)–(c) for the asymmetric series gap in microstrip lines. This set of graphs provides results for a plastic substrate of low dielectric constant, namely $\epsilon_s=2.35$, and a thickness of $h_s=0.635$ mm. Since the tendency of the curves is the same as for the configuration discussed in Fig. 4, a detailed interpretation of the results is left to the interested reader. The numerical values of the gap and stray capacitances are, at least for tight coupling, considerably lower than for the substrate of of high dielectric constant which has been discussed in Fig. 4.

Finally, for completeness, a somewhat different case is treated in Fig. 6. The substrate again is of the plastic type with low dielectric constant of $\epsilon_s = 2.35$. The substrate thickness is now $h_s = 0.79$ mm. In contrast to the cases investigated before, the width w_1 of strip 1 is kept constant at $w_1 \cong 3h_s$ which is equivalent to a characteristic impedance of strip 1 of approximately 50 Ω. With the width w_1 of strip 1 increased by a factor of 3 compared to Fig. 5, the gap capacitance C_g and the stray capacitance C_{s1} should be larger by about the same factor as is indeed confirmed by inspection of the figures. The change-over of the stray element C_{s1} to inductive values does not occur in the range of widths w_2 examined in Fig. 6(b) due to the reduction of the line inductance of line 1 with increased width, and as a result of the larger line capacitance. Likewise, the pronounced capacitive behavior of the structure is clearly visible from the relatively high values of stray capacitance C_{s2} in Fig. 6(c).

III. Conclusion

The elements of the equivalent pi-circuit of the asymmetrical series gap in microstrip and suspended substrate lines have been computed numerically for a wide range of geometries and two representative substrate materials. The results show a behavior of the asymmetric gap which is different from the case of equal widths of the involved strips and which is seen to tend towards that of the impedance step if end-to-end coupling between the strips is tight. Physical understanding, agreement with the results of other authors for the case of equal widths, as well as the asymptotic behavior and values of the computed equivalent circuit data confirm their validity. The information presented in graphical form was not available up to now and is thought to be useful for MIC design and measurement purposes.

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Evaluation of the Integrals Occurring in the Study of Circular Microstrip Disk

KOHEI HONGO AND MASARU TAKAHASHI

Abstract —**We have derived rigorous series expressions for the integrals which appear in the study of a circular microstrip disk separated from the grounded dielectric substrate when the problem is formulated by the method of dual integral equation or using Kobayashi potentials. The validity of the approximate expression of the integrals for small separation is verified numerically.**

I. Introduction

Recently, the electrostatic problem of a circular parallel plate condenser filled with dielectric has been of interest again because it has application as components of microstrip and antenna circuits. Bokar and Yang [1] studied this problem using the dual integral equation, and shortly later, Chew and Kong [2], starting from a similar approach, have derived the limiting values of capacitance as the separation approaches zero. The method of dual integral equation is closely related with the method of Kobayashi potential [3] developed by Kobayashi and Nomura [4]. The basic idea of these methods is to reduce the problem to a set of linear equations, and the crucial point of the methods is to calculate the matrix elements which are given by infinite integrals, including Bessel functions, as integrands. An analytical calculation of these kinds of integrals was first carried out by Nomura associated with a circular parallel capacitor located in an empty space [5]. But it is found that his result was incomplete, although his approach is very general. Recently, Chew and Kong [2] met with similar integrals which give matrix elements and tried to derive approximate solutions using much mathematical manipulation. However, their calculation was restricted to the first two terms of the power series expansion of the integrals.

The purpose of this article is to show that the infinite integrals can be calculated analytically through a rather straight-forward manner, which are valid for both large separation and small separation. When the separation is very small, approximate ex-

Manuscript received January 26, 1982; revised March 29, 1982.

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